Cost-Benefit Analysis, Distributional Weights, and Institutions

Matthew D. Adler
Duke University
Overview

• The theory of SWFs (Adler, *Well-Being and Fair Distribution*, OUP 2012)

• CBA with distributional weights to mimic an SWF (Adler, “Cost-Benefit Analysis and Distributional Weights,” 2013)

• Institutional Objection 1: Doesn’t the specification of an SWF/weights involve contestable value choices?

• Objection 2: Why not the tax system?
The SWF Framework

- The SWF approach ranks outcomes as a function of their corresponding utility vectors.
- Formally, the approach adopts some rule $R$ for ranking vectors. It then says:
  
  outcome $x$ is morally at least as good as $y$ ($x \succeq^M y$) iff
  
  $(u_1(x), u_2(x), \ldots, u_N(x)) R (u_1(y), u_2(y), \ldots, u_N(y))$

  $u(.)$ needs to be interpersonally comparable.
The SWF concept in economics

- Theoretical welfare economics: Bergson/Samuelson; Arrow’s theorem; Sen’s response
- Optimal tax theory: Mirrlees
- Currently used in various subfields of economics, including environmental econ. E.g., climate change models: Stern, Nordhaus, etc.
- Substantial scholarly literature going back to 1950s shows how to use weighted CBA to mimic SWF. Weighted CBA used for a time at World Bank and currently recommended in U.K.
Utilitarian and Prioritarian SWFs

- Two widely used SWFs are utilitarian and prioritarian (specifically isoelastic/Atkinson SWFs).
- The utilitarian SWF: $x \succeq_M y$ iff $\sum u_i(x) \geq \sum u_i(y)$
- The continuous prioritarian SWF: $x \succeq_M y$ iff $\sum g(u_i(x)) \geq \sum g(u_i(y))$, with $g(.)$ strictly increasing and concave
- The isoelastic/Atkinson SWF: $x \succeq_M y$ iff $(1-\gamma)^{-1} \sum u_i(x)^{1-\gamma} \geq (1-\gamma)^{-1} \sum u_i(y)^{1-\gamma}$, with $\gamma > 0$ an inequality-aversion parameter.
The prioritarian SWF

- $g(u_j)$
- $g(u_j - \Delta u)$
- $g(u_i + \Delta u)$
- $g(u_i)$

transformed utility, $g(u)$

utility, $u$

$u_i$, $u_i + \Delta u$, $u_j - \Delta u$, $u_j$
Possible axioms for an SWF

• Pareto superiority: \((3, 4, 10, 13) \succ (3, 4, 10, 12)\)

• “Anonymity”: \((7, 12, 4, 60) \sim (12, 60, 4, 7)\)

• Pigou-Dalton: \((3, 6, 8, 12) \succ (3, 4, 10, 12)\)

• Separability:
  \[(7, 100, 100, 7) \succeq (4, 100, 100, 12) \text{ iff }\]
  \[(7, 7, 7, 7) \succeq (4, 7, 7, 12)\]

• Continuity: If \((1, 3, 50000, 50000) \succ (1, 3, 6, 8)\), then \((1, 3 \pm \varepsilon, 50000, 50000) \succ (1, 3, 6, 8)\) for \(\varepsilon\) sufficiently small

• Ratio rescaling invariance:
  \[(10, 12, 17, 20) \succ (10, 10, 20, 20) \text{ iff } (50, 60, 85, 100) > (50, 50, 100, 100)\]
The Universe of Paretian, Anonymous SWFs

Prioritarian SWFs

Continuous

Prioritarian SWFs

Ratio-rescaling invariant?

Yes

No

- Atkinson SWF

- Other Continuous Prioritarian SWFs (e.g., Negative Exponential SWF)

Yes

No

- Leximin SWF
- Prioritarian SWF with Lexical Threshold

Satisfy the continuity axiom?

Yes

No

Satisfy the separability axiom?

Yes

No

- Utilitarian SWF
- Sufficientist SWF

- Rank-weighted SWF
How to specify $\gamma$

• This is an *ethical* parameter, capturing the decisionmaker’s moral/ethical views. Problematic to try to estimate empirically.

• A larger value of $\gamma$ means a more inequality-averse SWF. With $\gamma = 0$, SWF becomes utilitarian. With $\gamma = \infty$, SWF becomes leximin

• Leaky bucket question. Poor is at utility level $U$, while Rich is at level $KU$. If we reduce Rich’s well-being by $\Delta u$, and increase Poor’s by $f\Delta u$, $0 < f < 1$, what is the smallest value of $f$ such that this remains a moral improvement? For given $\gamma$, $f = (1/K)^\gamma$

• Equalization question. Is it an improvement to move from $(U, U^*)$ to $(U^+, U^+)$, where $U + U^* > 2U^+$?
Utilitarian and Isoelastic Distributive Weights

- Individual i’s status quo bundle is \((c_i^s, p^s, a_i^s)\) and in outcome \(x\) is \((c_i^x, p^x, a_i^x)\). Consumption, prices, nonmarket attributes.
- Her equivalent variation for \(x\), \(\Delta c_i^x\), is such that she is indifferent b/w \((c_i^s + \Delta c_i^x, p^s, a_i^s)\) and her bundle in \(x\).
- For small changes, the utilitarian SWF can be approximated by \(\sum MU_i \times \Delta c_i^x\), with \(MU_i\) the status quo marginal utility of income for individual \(i\).
- For small changes, the isoelastic SWF can be approximated by \(\sum MMVU_i \times MU_i \times \Delta c_i^x\), with \(MU_i\) the status quo marginal utility of income for individual \(i\), and \(MMVU_i\) (“marginal moral value of utility”) equaling \(u(c_i^s, a_i^s)^{-\gamma}\). This is where \(\gamma\) parameter appears.
The Functional Form of Utility

• $u(.)$ is defined by conditional vNM utility of consumption $h^a(.)$ at given level of $a$, plus ordinary tradeoffs between consumption and non-market attributes

• A simplifying assumption: $h(.)$ is invariant to $a$

• A further simplifying assumption: $h(.)$ is CRRA, i.e., $h(c) = (1-\lambda)^{-1}c^{1-\lambda}$. Much work on distributional weights assumes this form. CRRA generally popular. $\lambda$ is parameter of individuals’ personal preferences over consumption gambles, while $\gamma$ for isoelastic SWF is parameter of social planner’s moral/ethical preferences.
### Formulas for Weights

<table>
<thead>
<tr>
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<th>General Case</th>
<th>Invariance</th>
<th>Invariance plus CRRA</th>
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</thead>
<tbody>
<tr>
<td><strong>Utilitarian Weights</strong></td>
<td>Sum of $MU_i \times \Delta c_i^x$</td>
<td>Sum of $MU_i \times \Delta c_i^x$</td>
<td>Sum of $MU_i \times \Delta c_i^x$</td>
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<tr>
<td></td>
<td>$MU_i = f(c_i^s, a_i^s)$</td>
<td>$MU_i = h'(c_i^s) \times m(a_i^s)$</td>
<td>$MU_i = (c_i^s)^{-\lambda} \times m(a_i^s)$</td>
</tr>
<tr>
<td><strong>Isoelastic Weights</strong></td>
<td>Sum of $MMVU_i \times MU_i \times \Delta c_i^x$</td>
<td>Sum of $MMVU_i \times MU_i \times \Delta c_i^x$</td>
<td>Sum of $MMVU_i \times MU_i \times \Delta c_i^x$</td>
</tr>
<tr>
<td>$\gamma &gt; 0$ is coefficient of inequality aversion</td>
<td>$MU_i = f(c_i^s, a_i^s)$</td>
<td>$MU_i = h'(c_i^s) \times m(a_i^s)$</td>
<td>$MU_i = (c_i^s)^{-\lambda} \times m(a_i^s)$</td>
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<tr>
<td></td>
<td>$MMVU_i = u(c_i^s, a_i^s)^{-\gamma}$</td>
<td>$MMVU_i = u(c_i^s, a_i^s)^{-\gamma}$, with $u(c_i^s, a_i^s) = h(c_i^s) \times m(a_i^s) + k(a_i^s)$</td>
<td>$MMVU_i = u(c_i^s, a_i^s)^{-\gamma}$, with $u(c_i^s, a_i^s) = h(c_i^s) \times m(a_i^s) + k(a_i^s)$ and $h(c) = (1-\lambda)^{-1} c^{1-\lambda}$</td>
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Distributive Weights and VSL

• A simple model. Each individual in status quo has probability $p_i$ of surviving period and consumption $c_i$. $h_{\text{survive}}(c) = u(c, \text{survive}); \ h_{\text{die}}(c) = u(c, \text{die}). \ h_{\text{survive}}(c) > h_{\text{die}}(c); \ h'_{\text{survive}}(c) > h'_{\text{die}}(c) \geq 0; \ h''_{\text{survive}}(c) \leq 0, h''_{\text{die}}(c) \leq 0.$

• $U_i = p_i h_{\text{survive}}(c_i) + (1-p_i) h_{\text{die}}(c_i). \ VSL_i = \frac{\partial U_i}{\partial p_i} \left/ \frac{\partial U_i}{\partial c_i} \right. = \frac{h_{\text{survive}}(c_i) - h_{\text{die}}(c_i)}{p_i h'_{\text{survive}}(c_i) + (1-p_i) h'_{\text{die}}(c_i)}$

• If policy changes individual’s survival probability by $\Delta p^p_i$ and consumption by $\Delta y^p_i$, his equivalent variation $\Delta c^p_i$ is approximately $\Delta y^p_i + VSL_i \Delta p^p_i$

• I assume, specifically, that $h_{\text{survive}}(.)$ is CRRA; that $h'_{\text{die}}(.)$ is 0; there is subsistence level of consumption $c^*$ such that $h_{\text{survive}}(c^*) = h_{\text{die}}(c)$ for any $c$
Rich have annual income of $100K, and annual all-cause fatality risk of 0.005, while Poor have annual income of $20K and annual all-cause fatality risk of 0.01. Four hypothetical policies, all with 1/100,000 risk reduction. (1) Uniform Risk Reduction and Cost Incidence (both groups receive risk reduction and pay same costs); (2) Uniform Risk Reduction and Redistributive Incidence (both groups receive risk reduction and Rich pay costs); (3) Concentrated Risk Reduction and Cost Incidence (Poor receive risk reduction and pay costs); (4) Regressive Risk Transfer (risk transferred from Rich to Poor, e.g., siting decision)

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<thead>
<tr>
<th></th>
<th>$\lambda = 0.5$</th>
<th>$\lambda = 0.5$</th>
<th>$\lambda = 1$</th>
<th>$\lambda = 1$</th>
<th>$\lambda = 2$</th>
<th>$\lambda = 2$</th>
<th>$\lambda = 3$</th>
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<td>Sub=5000</td>
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<tr>
<td>$VSL_{Rich}$</td>
<td>$180,905$</td>
<td>$156,059$</td>
<td>$462,831$</td>
<td>$301,079$</td>
<td>$9,949,749$</td>
<td>$1,909,548$</td>
<td>$502$ mill</td>
<td>$20,050,251$</td>
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<tr>
<td>$VSL_{Poor}$</td>
<td>$31,369$</td>
<td>$20,202$</td>
<td>$60,520$</td>
<td>$28,006$</td>
<td>$383,838$</td>
<td>$60,606$</td>
<td>$4,030,303$</td>
<td>$151,515$</td>
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<tr>
<td>$VSL_{Rich}/VSL_{Poor}$</td>
<td><strong>5.8</strong></td>
<td><strong>7.7</strong></td>
<td><strong>7.6</strong></td>
<td><strong>10.8</strong></td>
<td><strong>25.9</strong></td>
<td><strong>31.5</strong></td>
<td><strong>124.7</strong></td>
<td><strong>132.3</strong></td>
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<tr>
<td>$U-\text{Weight}<em>{Poor}/U-\text{Weight}</em>{Rich}$</td>
<td>2.2</td>
<td>2.2</td>
<td>5</td>
<td>5</td>
<td>24.9</td>
<td>24.9</td>
<td>124.4</td>
<td>124.4</td>
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<td>$\gamma = 0.5$</td>
<td>3.6</td>
<td>4.2</td>
<td>6.2</td>
<td>7.3</td>
<td>25.5</td>
<td>28.1</td>
<td>124.8</td>
<td>128.6</td>
</tr>
<tr>
<td>$\gamma = 0.8$</td>
<td>5.8</td>
<td>7.8</td>
<td>7.7</td>
<td>10.8</td>
<td>26.1</td>
<td>31.7</td>
<td>125.3</td>
<td>133</td>
</tr>
<tr>
<td>$\gamma = 2$</td>
<td>15.1</td>
<td>27.1</td>
<td>11.9</td>
<td>23.5</td>
<td>27.3</td>
<td>40.3</td>
<td>126.2</td>
<td>142.2</td>
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<tr>
<td>Uniform Risk Reduction and Cost Incidence</td>
<td>Concentrated Risk Reduction and Cost Incidence</td>
<td>Regressive Risk Transfer</td>
<td></td>
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<tr>
<td>Maximum per capita cost imposed uniformly on Rich and Poor</td>
<td>Maximum per capita cost imposed on Rich</td>
<td>“Yes” if the transfer is assigned a positive sum of monetary equivalents, “No” if it is assigned a negative sum, “Neutral” if assigned a zero sum</td>
<td></td>
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<td></td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>CBA w/o weights</th>
<th>$51.67</th>
<th>$103.33</th>
<th>$3.84</th>
<th>Yes</th>
</tr>
</thead>
<tbody>
<tr>
<td>CBA with utilitarian weights</td>
<td>$7.54</td>
<td>$194.97</td>
<td>$3.84</td>
<td>Yes</td>
</tr>
<tr>
<td>CBA with isoelastic weights, $y = 0.5$</td>
<td>$7.45</td>
<td>$197.21</td>
<td>$3.84</td>
<td>Yes</td>
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<tr>
<td>CBA with isoelastic weights, $y = 1$</td>
<td>$7.37</td>
<td>$199.50</td>
<td>$3.84</td>
<td>No</td>
</tr>
<tr>
<td>CBA with isoelastic weights, $y = 2$</td>
<td>$7.22</td>
<td>$204.23</td>
<td>$3.84</td>
<td>No</td>
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<tr>
<td>CBA with population average VSL</td>
<td>$42.33</td>
<td>$84.67</td>
<td>$42.33</td>
<td>Neutral</td>
</tr>
</tbody>
</table>

**Note:** $\lambda = 2$, subsistence level = 1000
Objection 1 to Distributive Weights: Value Choices

• Does the choice of an SWF involve value choices? Yes!! The SWF framework is a methodology for moral/ethical decisionmaking, and specifying this methodology involves value choices: (a) The nature and measurement of well-being (preferences vs. happiness vs. capabilities; how to handle interpersonal comparisons); (b) functional form of the SWF; (c) γ parameter for Atkinson SWF.

• It is misconceived to assume that society typically “shares” a single SWF. Rather, a given SWF embodies a particular, contestable, ethical view, which will be held by some individuals (a political party?) but rejected by others.

• Is it legally permissible for regulators (within their zone of statutory discretion) to act upon (contestable) ethical views? Difficult question of jurisprudence. Hart/Dworkin etc. At a minimum, executive-agency regulators can implement the President’s preferred SWF. **Note:** Traditional CBA itself (Kaldor-Hicks) is deeply controversial (outside this room), and has largely been given effect by Presidential order. While the scope of the Prez’s EO power is contested, a weighted-CBA EO analogous to EO 12866 would surely be legally legitimate if EO 12866 is!
Objection 2: Why not the tax system?

Assume that regulatory option R is preferred by an SWF, while R* is preferred by unweighted CBA. If perfect lump-sum taxes are possible (all individual attributes are observable, and taxes have zero administrative costs), there exists a tweak T* to the tax system such that R*-plus-T* is Pareto superior to R. Shouldn’t the regulator choose R*? (Assume, specifically, an executive-agency regulator implementing the President’s preferred SWF.)

Case I, “Divided government”: The President’s preferred SWF is not fully shared by Congress, which controls the tax system. In this case, depending on the makeup of Congress, the regulator may rationally predict that R* would be left in place, or would be accompanied by a different tweak T+ such that R*-plus-T+ is dispreferred by the Prez’s SWF to R.
Objection 2: Why not the tax system?

- **Case 2, “Unified government”:** The President and Congress have the same SWF.
- **Kaplow:** Assumes the standard optimal-tax set up, with individual “ability”/labor unobservable, but individuals have a common utility function; this is weakly separable in labor, i.e., $u(.) = u(v(c,a), l)$; and all of the $a$ attributes are observable. Then if $R^*$ is preferred by unweighted CBA to $R$, there exists a tweak $T^*$ to the tax system such that $R^*$-plus-$T^*$ is Pareto superior to $R$, even taking account of labor supply.
- This is an elegant model, but many of its assumptions may be violated empirically: (1) hidden $a$ attributes (e.g., health); (2) heterogeneous preferences; (3) salience and labor supply; (4) paternalism and in-kind benefits; (5) administrative costs of tax tweaks.
- Even with labor supply fixed, if there are hidden $a$ attributes, the regulator may estimate their population distribution (thus ascertain that $R^*$ is preferred by CBA to $R$), but not be able to key taxation to $a$ attributes. Health example.
Conclusion

• An SWF involves contestable value choices, but so does traditional CBA. The Prez gets to make those value choices, up to the limits of clear statutory meaning (Chevron). It is legally permissible for an executive-agency regulator (how about independents?) to act on the Prez’s preferred SWF within her zone of statutory discretion. While EO 12866 does not require (does it allow?) distributive weights, the Prez could certainly amend 12866 to require that—even though his SWF might not be “society’s” (there is no such thing). In any case where the SWF prefers R while unweighted CBA prefers R*, the regulator should consider whether R* might be combined with politically feasible tax changes T* so that the SWF prefers R*-plus-T* to R. But such a T* may not exist, either because of ideological differences b/w the Prez and Congress, or because the combined assumptions of Kaplow’s model do not hold.
Why SWFs need I-P Comparability

<table>
<thead>
<tr>
<th></th>
<th>Original</th>
<th>Renumbering 1</th>
<th>Renumbering 2</th>
<th>Renumbering 3</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>x</td>
<td>y</td>
<td>Diff.</td>
<td>x</td>
</tr>
<tr>
<td>Jim</td>
<td>10</td>
<td>11</td>
<td>1</td>
<td>200</td>
</tr>
<tr>
<td>Sue</td>
<td>30</td>
<td>25</td>
<td>-5</td>
<td>60</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>36</td>
<td>260</td>
<td>204</td>
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- Renumbering 1 is **individual-specific ratio transformation** (preserves only intrapersonal info). Renumbering 2 is **common affine transformation** (preserves intra- and interpersonal levels and differences). Renumbering 3 is **common ratio transform**. Utilitarian SWF invariant to Renumberings 2 and 3, Isoelastic only to 3.
The Mathematics of Utilitarian Weights

\[ w(x) - w(s) = \sum_{i=1}^{N} u(c_i^x, p_i^x, a_i^x) - u(c_i^s, p_i^s, a_i^s) = \]

\[ \sum_{i=1}^{N} u(c_i^s + \Delta c_i^x, p_i^s, a_i^s) - u(c_i^s, p_i^s, a_i^s) \]

This is approximated by:

\[ \sum_{i=1}^{N} \frac{\partial u(c_i^s, p_i^s, a_i^s)}{\partial c} \Delta c_i^x \]
The Mathematics of Isoelastic Weights

\[ e(x) - e(s) = \sum_{i=1}^{N} (1 - \gamma)^{-1} [u(c_i^s + \Delta c_i^x, p^s, a_i^s)^{1-\gamma} - u(c_i^s, p^s, a_i^s)^{1-\gamma}] \]

This is approximated by:

\[ \sum_{i=1}^{N} u(c_i^s, p^s, a_i^s)^{-\gamma} \frac{\partial u(c_i^s, p^s, a_i^s)}{\partial c} \Delta c_i^x \]
Distributive Weights under Uncertainty

- **Utilitarian SWF**: $EMU_i \times \Delta c_i^P$

\[
\frac{1}{N} \sum_{i=1}^{N} \sum_{z \in Z} \pi(z) u(c_i^{s,z} + \Delta c_i^P, p^{s,z}, a_i^{s,z}) - \frac{1}{N} \sum_{i=1}^{N} \sum_{z \in Z} \pi(z) u(c_i^{s,z}, p^{s,z}, a_i^{s,z})
\]

approximated by

\[
\frac{1}{N} \sum_{i=1}^{N} \sum_{z \in Z} \pi(z) \frac{\partial u(c_i^{s,z}, p^{s,z}, a_i^{s,z})}{\partial c} \Delta c_i^P
\]

- **Ex Ante Isoelastic SWF**: $MMVEU_i \times EMU_i \times \Delta c_i^P$

\[
(1-\gamma)^{-1} \left( \frac{1}{N} \sum_{i=1}^{N} \left( \sum_{z \in Z} \pi(z) u(c_i^{s,z} + \Delta c_i^P, p^{s,z}, a_i^{s,z}) \right) \right)^{1-\gamma} - (1-\gamma)^{-1} \left( \frac{1}{N} \sum_{i=1}^{N} \left( \sum_{z \in Z} \pi(z) u(c_i^{s,z}, p^{s,z}, a_i^{s,z}) \right) \right)^{1-\gamma}
\]

approximated by

\[
\left( \frac{1}{N} \sum_{i=1}^{N} \left( \sum_{z \in Z} \pi(z) u(c_i^{s,z}, p^{s,z}, a_i^{s,z}) \right) \right)^{-\gamma} \left( \frac{1}{N} \sum_{i=1}^{N} \sum_{z \in Z} \pi(z) \frac{\partial u(c_i^{s,z}, p^{s,z}, a_i^{s,z})}{\partial c} \right) \Delta c_i^P
\]
SWFs under Uncertainty

\[ e = \sum v_i, \text{ } A \text{ and } B \text{ are equiprobable states} \]

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<td>A</td>
<td>4</td>
<td>9</td>
<td>6.5</td>
</tr>
<tr>
<td>B</td>
<td>0</td>
<td>4</td>
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<td>3.5</td>
<td>3.5</td>
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<tr>
<td>B</td>
<td>3.5</td>
<td>3.5</td>
<td>3.5</td>
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</table>

\[ e = 2 \quad 5 \]

Expected \( e = 3.5 \)

(“ex post” approach)

\[ e \text{ applied to the vector of expected utilities} = 3.96 \]

(“ex ante” approach)

\[ e \text{ applied to the vector of expected utilities} = 3.74 \]