JOINT COST-BENEFIT ANALYSIS OF (RAIL TRACK) MAINTENANCE AND RENEWAL: THE ROLE OF QUALITY OF SERVICE

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OUTLINE

1. Classical approach to rail infrastructure maintenance cost analysis and its drawbacks

2. A new approach inspired by technical analysis:
   2.1. The model: mathematical formulation of a 3-phase model
   2.2. Maximization procedure and 3-régime solution
   2.3. Consequences for infrastructure charging
   2.4. Econometric test methodology: Box-Cox model with nonspherical residuals
   2.5. Calibration 0: are there A-B-C phases in actual maintenance expenditure data?
   2.6. With doubly censored Phase B samples, 3 relationships are studied

3. Conclusions and desirable further developments
1. THE CLASSICAL APPROACHES...

- Establish relations between total yearly maintenance cost and several drivers, among which:
  - Technical characteristics of the rail link
  - Traffic(s) on the rail link

- Are central to the most recent comprehensive work on this subject by the European Union CATRIN (2009) multinational research consortium

**DRAWBACKS**

Do not take into account the objective of maintenance, i.e. quality of service, or assume that the quality of service is kept constant.

Rarely take into account the fact that maintenance depends on cumulated traffic.

Do not properly account for renewal and treat it as an independent issue.
2. A NEW APPROACH: INSPIRED BY TECHNICAL ANALYSIS

- After a renewal/regeneration is carried out, quality of service is high
- Progressively, quality of service decreases due primarily to traffic damages, and can be increased by current maintenance
- But the maintenance level necessary to maintain quality of service increases over time becomes higher because damages depend on cumulated traffic
- And, at some point $T$, it is better to renew the track than to continue current maintenance

Take the example of a certain index of (longitudinal) track degradation NL and the decreasing impact of tamping maintenance on it (between regenerations):

- $\text{/ : track degradation index NL}$
- $\downarrow$ : effect of tamping on NL
- $\Rightarrow$ Minimum service level

<table>
<thead>
<tr>
<th>Duration $T$ between ballast regenerations</th>
<th>$\text{NL mini}$</th>
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<tbody>
<tr>
<td>$\text{Minimum service level}$</td>
<td>-----------------</td>
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</tbody>
</table>
2.1. Mathematical formulation of a 3-phase model

- \( dS = h(K, Q(t), t) \star [u(t) - f(K, Q(t), q(t), t)] \star dt \)

Money value of the quality of service: \( q(t) \star g(S(t)) = -\alpha q(t) e^{-\lambda S(t)} \)

Optimisation function:

\[
M^* = \max_{u(t), T} \left\{ \sum_{i=0}^{\infty} \left[ \left( \int_{T_i}^{T_{i+1}} [-u(t) + q(t)g(S(t))] e^{-jt} dt - De^{-jT_{i+1}} \right) e^{-jT_i} \right] \right\}
\]

Such that:

\( dS = h(K, Q(t), t) \star [u(t) - f(K, Q(t), q(t), t)] \star dt \)

And: \( 0 \leq u(t) \leq m \)

- Under the simplifying assumption that traffic \( q(t) \) is constant over time and denoted by \( q \), optimal regenerations will be regularly spaced at some interval \( T \) and the problem becomes:

\[
M^* = \max_{u(t), T} \left\{ \left( \int_{0}^{T} [-u(t) - \alpha q e^{-\lambda S(t)}] e^{-jt} dt - De^{-jT} \right) \frac{1}{1 - e^{-jT}} \right\}
\]

\[
= \max_{u(t), T} \left\{ [J(u(t), T) - De^{-jT}] \frac{1}{1 - e^{-jT}} \right\}
\]

- \( t \): time
- \( q(t) \): traffic density
- \( Q(t) \): traffic cumulated from 0 to \( t \)
- \( \frac{dQ(t)}{dt} = q(t) \): relation betw. \( Q \) and \( q \)
- \( K \): Technical characteristics of the link (max. speed, number of sleepers...)
- \( u(t) \): maintenance
- \( S(t) \): service quality
- \( T_i \): successive renewal times
- \( D \): renewal cost assumed constant and independent of other variables
- \( j \): discount rate
2.2. Optimization and 3-régime solution

- First optimize maintenance $u$ for a given renewal time $T$
- Second optimize $T$
- Hamiltonian:
  \[ H = [-u(t) - q(t) S(t)^{-\lambda}] e^{-jt} + y(t) \left[ h(K, Q(t), t) * [u(t)] - f(K, Q, q(t), t) \right] \]
- Pontryagin principle:
  \[ \max_u H = \max_u \left\{ u(t) \left[ h(K, Q, t) y(t) - e^{-jt} \right] \right\} \]
  \[ H_S + \dot{y} = 0 \]
  \[ H_y = \frac{dS}{dt} \]
Max $H = \max_u \left\{ u(t) \ast \left[ h(K,Q,t) \ast y(t) - e^{-j^t} \right] \right\}$

three possible phases:

Phase A: $h(K,Q,t) \ast y(t) - e^{-j^t} < 0$. With $u(t)=0$ over this interval I, we have:
- $S(t)$ found by integration of $dS/dt = h(K,Q,t) \ast u(t) - f(K,Q,q,t)$: it is deduced, the phase starting at moment $t_f$ with $S(t_f)$ denoting service quality at that moment;
- $y(t)$ determined by $H_s + \dot{y} = 0$, which yields $y(t) = y(t_f) - \int_{t_f}^{t_f} \alpha q \lambda S(t)^{-\lambda-1} e^{-j^t} dv$.

Phase C: $h(K,Q,t) \ast y(t) - e^{-j^t} > 0$. With $u(t)=m$ over this interval III, we have:
- $S(t)$ by integration of $dS/dt = h(K,Q,t) \ast m - f(K,Q,q,t)$, the phase starting at time $t_m$;
- $y(t)$ determined by $H_s + \dot{y} = 0$, which yields $y(t) = y(t_m) + \int_{t_m}^{t} q \lambda e^{-\lambda S(t)} e^{-j^t} dv$.

Phase B: $h(K,Q,t) \ast y(t) = e^{-j^t}$. In reverse order now, we have over this central interval II:
- $y(t) = \left[ 1/h(K,Q,t) \right] e^{-j^t}$, by simple manipulation of the phase condition;
- $S_c(t)$, determined by $H_s + \dot{y} = 0$, and to be called cruising service quality:

\[
\dot{y} + H_s = \frac{-je^{-j^t}}{h(K,Q,t)} - \frac{\partial h}{\partial t} + \left[ \frac{\partial h}{\partial Q} \right] \left[ \frac{\partial Q}{\partial t} \right] e^{-j^t} + \alpha q \lambda S_c(t)^{-\lambda-1} = 0,
\]

and

(8) \[
\log[S_c(t)] = \frac{1}{\lambda + 1} \left\{ \log \left[ \frac{1}{h + \frac{h'}{j} \lambda} \right] + \log \left[ \frac{1}{h + \frac{1}{j} \lambda} \right] - \log \left[ \frac{1}{h + \frac{1}{j} \lambda} \right] \right\},
\]

with $u(t)$ then given by:

(9) \[
u(t) = f(K,Q,q,t) + \frac{1}{h(K,Q,t)} \frac{dS_c(t)}{dt}.
\]
Typical evolution of service quality and maintenance expense for $T=35$

<table>
<thead>
<tr>
<th>A. Service quality (initial value equals 250)</th>
<th>B. Maintenance expenditure ($nil$ close to $T=35$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>![Quality of service graph]</td>
<td>![Maintenance expenditure graph]</td>
</tr>
</tbody>
</table>

And we also introduce uncertainty...(see paper).
2.3. Consequences for marginal social cost pricing

The effect of quality of service is not negligible

A kind of Mohring effect for flows with small traffics

<table>
<thead>
<tr>
<th>Traffic</th>
<th>Revenue from standard marginal charge</th>
<th>Revenue from new optimal charge</th>
</tr>
</thead>
<tbody>
<tr>
<td>400</td>
<td>1,509</td>
<td>1,630</td>
</tr>
<tr>
<td>300</td>
<td>1,080</td>
<td>1,140</td>
</tr>
<tr>
<td>200</td>
<td>0,583</td>
<td>0,588</td>
</tr>
<tr>
<td>100</td>
<td>0,173</td>
<td>0,155</td>
</tr>
<tr>
<td>50</td>
<td>0,057</td>
<td>0,038</td>
</tr>
</tbody>
</table>
2.4. Econometric test methodology: Box-Cox model with non-spherical residuals (heteroskedasticity and spatial correlation)

\[ y_{t}^{(\lambda_v)} = \beta_0 + \sum_k \beta_k X_{kt}^{(\lambda_k)} + u_t, \]

\[ u_t = \left[ \exp \left( \sum_m \delta_m Z_{mt}^{(\lambda_m)} \right) \right]^{1/2} v_t, \]

\[ v_t = \sum_{\ell=1}^{2} \rho_{\ell} \left( \sum_{n=1}^{n=T} \tilde{r}_{\ell,m} v_n \right) + w_t. \]

where

\[ \text{Var}_{v}^{(\lambda_v)} \equiv \begin{cases} (\text{Var}_{v})^{\lambda_v} - 1)/\lambda_v, & \lambda_v \neq 0, \\ \ln(\text{Var}_{v}), & \lambda_v \to 0, \end{cases} \]

\( \tilde{r}_{\ell,m} \) : element of Boolean matrix \( \tilde{R}_{\ell} \) normalized

[if \( \rho_{\ell} > 0 \) (substitutes); if \( \rho_{\ell} < 0 \) (complements)].

Or, in matrix form:

\[ v = \sum_{\ell=1}^{2} \rho_{\ell} \tilde{R}_{\ell} v + w, \]

\[ \tilde{R}_{\ell} = \pi_{\ell} \left[ I - (1 - \pi_{\ell}) \tilde{R}_{\ell} \right]^{-1} \tilde{R}_{\ell}, \quad (0 < \pi_{\ell} \leq 1), \]
2.5. Calibration 0: are there 3 phases in actual maintenance expenditure data?

\[ u_t = f(K, Q, q, t) + \frac{1}{h(K, Q, t)} \left[ Sc_t - Sc_{t-1} \right] + a(K, Q, t) \left[ S_{t-1} - Sc_{t-1} \right] + \varepsilon_{it} \]

We take only the first term and focus on \( Q \)

\[ u_t = f(K, Q, q, t) + \varepsilon_{it} \]

and establish the presence of turning forms, the first implication of the model, by using up to 2 Box-Cox transformations on the same cumulative traffic variable \( Q \):

\[ f(Q_t) = \beta_{Q1} Q_t^{(\lambda_{Q1})} + \beta_{Q2} Q_t^{(\lambda_{Q2})} \]

which yields quadratic forms as special cases of a more general forms with asymmetric slopes

**Sign conditions for a maximum or a minimum with two BCT on the same variable Q**

<table>
<thead>
<tr>
<th>CASE</th>
<th>( \beta_1 )</th>
<th>( \beta_2 )</th>
<th>( \lambda_1 - \lambda_2 )</th>
<th>( \beta_1(\lambda_1 - \lambda_2) ) or ( \beta_2(\lambda_2 - \lambda_1) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \cap )</td>
<td>Maximum 1</td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( \cup )</td>
<td>Minimum 1</td>
<td>+</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>( \cap )</td>
<td>Maximum 2</td>
<td>-</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>( \cup )</td>
<td>Minimum 2</td>
<td>-</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>

Then we remove Phase A and C observations, censoring the data to keep Phase B observations
2.6. With doubly censored Phase B samples, three relationships are studied:

A. A target service $S_{ct}$ function (with a panel) is estimated and $E(S_{ct})$ used for 2 new terms

<table>
<thead>
<tr>
<th>Relationship</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Target Service adjustment</td>
<td>$E(S_{ct}) - E(S_{ct-1})$</td>
</tr>
<tr>
<td>Trajectory Correction</td>
<td>$S_t - E(S_{ct-1})$</td>
</tr>
</tbody>
</table>

introduced to explain:

B. maintenance expenditures $u_t$ in addition to terms previously used (traffic, track characteristics)
   - they are statistically significant and of the right sign;

C. the evolution of the quality of service $(\Delta S)_t$
   - they are statistically significant and of the right sign;
3. Conclusions and desirable further developments

- A model theoretically linking
  - Current maintenance and renewal (only one model needed, not two)
  - Infrastructure maintenance expenses and quality of service

With 3 phases, in accordance with technical experience and implying a target service

With two new terms improving the standard explanation of maintenance expenses and allowing for an unprecedented explanation of the evolution of service. In conformity with the approach

  - Sophisticated flexible form econometric test results are in conformity with the new approach, but does the operator really optimize its behaviour?

- Possible extensions appear warranted:
  - Non constant traffic flows
  - Optimal timing of renewal in the presence of uncertainty
  - Better data on quality of service and on cumulated traffics by train type
  - Tests of other specifications for the traffic damage law

- Quality of service is finally put at the heart of the problem and its presence appears validated